**Homework 07: Quantum Dynamics II**

**PHYS550 – Quantum Mechanics I**

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***Additional Texts Referenced: Introduction to Quantum Mechanics, Griffiths and Schroeter***

**Problem 2.1**

*Consider the spin-precession problem discussed in the text. It can also be solved in the Heisenberg picture. Using the Hamiltonian:*

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*Write the Heisenberg equations of motion for the time-dependent operators Sx(t), Sy(t), and Sz(t). Solve them to obtain Sx,y,z as a function of time.*

The Heisenberg equation of motion is:



Where the operator A is understood to be in its time-dependent form. In our case, this can be written as



Where “i” is the index and can be x, y, or z. H is always known, so we replace it with its definition and get:



The commutator for the spin operators is known:. As should be expected: if i=j then the commutator would be zero. If i≠j then the result is some constant times the third spin operator. The same would be true in either the Heisenberg or Schrodinger picture since:



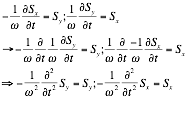
So in the case of i=z we already have an answer:



Which means it does not change with time so our time dependent Sz is the same as Sz in the Schrodinger picture. However, the other components will depend on time, specifically with the form



This is a rather simple system of differential equations which can be solved. See:



Where every solution is of the form



The only difference between Sx and Sy will be initial conditions, that is, what are they at the start? At t=0 they should be exactly equivalent to their forms in the Schrodinger picture; that is:



The actual state of the electron spin won’t matter. These would even be the case if the electron was pointing spin Sx down or anywhere else.



At t=0 we need these to equal the Schrodinger picture, which means a=1. Furthermore, we can pull out “i”.



This is clearly only possible if b=±i given the relation we already know of



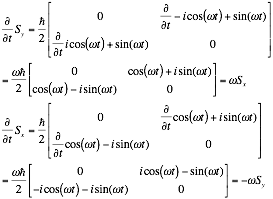
So:



Wherein we get the idea to split up the “+” and “-“ terms between the different locations in the matrix, giving us:



Since we did more than a few jumps to get here, a could way to check would be to confirm they are related derivatives of each other.



Confirmed! Thus, with our unchanging Sz, we have a final three-component operator of:

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Perhaps euler’s identity should have been used earlier, but it was easier to think through the trig functions and their behaviors.

**Problem 2.5**

*Let x(t) be the coordinate operator for a free particle in one dimension in the Heisenberg picture. Evaluate [x(t),x(0)]*

x(0) is just x, the normal position operator. x(t) is given by 

Since we don’t know our Hamiltonion H, we can’t pull x out since H could act on x for all we know. Strangely, it seems like an exact solution is already given in the textbook via equation 2.103



This is, however, given for the 3D case, but we have a 1D case. However, changing it to 1D changes nothing. Here is the entire proof redone with H=p2/2m rather than H=**p**2/2m. Equation 2.97a states that [xi,F(**p**)]=iћ(∂F/∂pi). In our case we are one dimensional so we can leave off the index. F, in our case, is going to be the Hamiltonian H. So we have:



Where use was made of x=iћ(∂/∂p) and p=-iћ(∂/∂x). We then solve the differential equation via general kinematics and get:



Where the “of 0” parts are added for clarity—x(0)=x and p(0)=p. Now if we take the commutator directly, we have:



Which is exactly what we started out with via equation 2.103, just with some of the extra steps filled in.

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Even though 2.103 is in three dimensions, there is no reason it should not also work for the one-dimensional case.

**Problem 2.8**

*Consider a free particle wave packet in one dimension. At t=0 it satisfies the minimum uncertainty relation*

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*In addition, we know*

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*Using the Heisenberg picture, obtain  as a function of t(t≥0) when  is given. (Hint: take advantage of the property of the minimum uncertainty wave packet you worked out in Chapter 1,* ***Problem 1.20****)*

The aforementioned minimum uncertainty wave packet relation from **Problem 1.20**:

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And the property we proved for this is

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Regardless, we seek ** which can always be written as ** but we know  is zero—not just at time equals zero, but at all times. We can argue this by logic: the Gaussian with initial zero momentum expectation is not “going anywhere” so it stays in the same place. But we can also prove it by adapting 2.101 to point out that x(t)=x(0)+p(0)t/m, which, if we replace everything with expectation values, is clearly always zero. Thus, we now simply seek to find ** as a function of time.

We can take 2.101 again to find a relation: by squaring it we end up with:



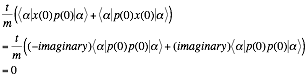
we seek to turn this into something in terms of x(0). Our first term already is in that form. Using the minimum uncertainty relation, we find that:

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Which puts our second term in the right form as well. Which just leaves the final part. We will use the **Problem 1.20** property to show that it cancels. First, let’s take our new term and write it out on bra-ket form.



This would be a simple matter if we could say x(0) = ∆x and p(0) = ∆p. Since ∆A=A-<A>, and <x> in this case is 0, the the **Problem 1.20** property is used to transform all the x-operators into p-operators.



Which then gives us

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Which is our answer.